high office you have intrusted to me. I must endeavour, so far as in me lies, to justify your choice; and at all events to show you, by my zeal for the progress and welfare of the Academy, that my best energies, such as they are, shall be devoted to your service. Our business here is not amusement, or relaxation, but the spread of learning, the communication of knowledge to each other, and to the public,—the interchange of that mutual encouragement, and sympathy, and support, which will enable us, each in his own department, to promote the great object of our Association,—the investigation and the discovery of Truth. Let a generous emulation to be foremost in this noble and glorious pursuit banish from our meetings all party spirit, all private differences. Our discussions will, I trust, be at all times conducted with manly freedom,—but even when we differ in opinion from each other, let us remember that the expression and calm discussion of such differences is one of the most important instruments for the discovery of Truth; and let our debates be an example of the philosophic spirit, which is most in accordance with the objects for which we are incorporated, which is, most agreeable also to the feelings of the polished gentleman and to the instincts of the enlightened Christian. Then may we hope that the meetings of the Academy may continue to be to others what they have already been to us, the means of forming deep and lasting friendships, the source of warm personal attachments, and of the highest intellectual enjoyment; and we may then hope for the more complete fulfilment of that noble aspiration, with which the accomplished Burrowes concluded his Preface to the first volume of our Transactions:—

"The GOD of Truth will look propitious on our labours, and a ray from Heaven shall light us to success."

Mr. Gilbert Sanders read a notice of some properties of solid figures revolving on axes in supports fixed at the surface level of fluids.

A sector of any solid figure which may be described by the revolution of any plane round an axis, if freely suspended by the axis on supports fixed at the surface level of any fluid,

and weighted and balanced till its descending force shall be equal to a power represented by half the specific gravity of the fluid,—it will lie on the surface of the fluid without any part being immersed; but if any of the fluid be afterwards withdrawn, the solid figure, or float, will descend in exact proportion to the quantity withdrawn, volume for volume, replacing by its own bulk the abstracted fluid, provided the quantity of fluid removed does not exceed the float in volume; and the float will again ascend in proportion as the abstracted fluid is restored. The float, so described, by its ascent and descent during the removal and replacement of the fluid, will maintain the surface level of the fluid at the original points.

To consider the foregoing proposition, let us suppose, in Fig. 1, MN a cistern of water, of which the surface level is

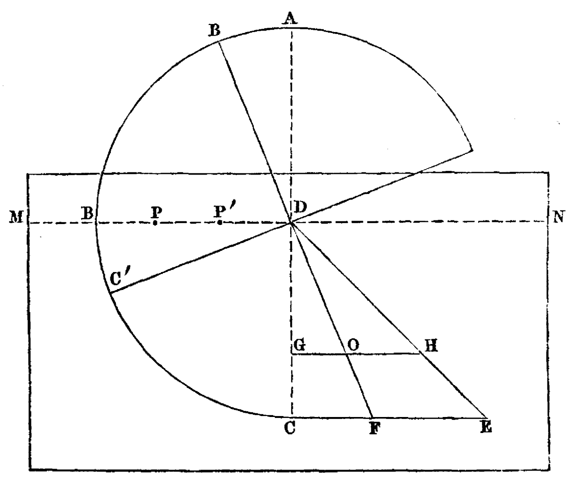

Fig. 1.

the line MN; let ABC be a sector of a semicylinder of any breadth, and the side AC perpendicular to the surface level MN. (Our remarks on the present occasion will be confined to the semicylinder, as that figure is the most suitable for our purpose.) Let D be the axis of the semi-cylindrical float,

turning freely on fixed supports at the surface of the water MN. Draw CE at right angles to DC, and equal to it. Suppose DC divided into any number of indefinitely small parts; the pressure on any part, as at C, will be as its depth from D, which is equal to the perpendicular CE; and similar perpendiculars, drawn from any other part, will be equal to the depth of such part, and the whole pressure on DC is represented by the triangle DCE. Bisect CE in F, and join DF; the centre of gravity of the triangle DCE is at two thirds of DF, from D at the point O, through which draw GH parallel to CE. The sum of all the perpendiculars, multiplied by their respective forces, is equal to the sum of all the forces multiplied by their mean distance, which is GO; and, therefore, the pressure may be considered as concentrated at O, and acting along the line GO, or at G, which is at two-thirds of DC from D, and, therefore, the force of the water pressing against the line DG, is expressed by two-thirds of DC, multiplied by the weight of a quantity of water represented by, or equal to, the triangle DCE. Now, it is evident that an equal pressure or weight acting perpendicularly at P, two-thirds of DB, from D, will balance the pressure at G.

Suppose the figure to have revolved about the axis D, till DC became DC′, and DB, DB′. The angles BDC′ and B′DA are equal, and their sines also equal; but the rotating power of any weight acting at any given point in BD is to its power at B′D as the sine of the angle made by BD with DA, and the pressure of the water on DC′ is as the sine of the equal angle BDC′, for the pressure on each of the parts into which DC was supposed to be divided is as their depths, or the perpendiculars let fall on them from the line BD, that is, as the sine of the angle BDC′; therefore, the pressure of the sum is likewise as the sine of BDC′, and consequently equal to the power of the weight at P on B′D, and they will balance each other in all positions, as the same may be proved of any position of the float.

Now, if the water sink below the level of BD, the pressure

on DC will be diminished and the balance disturbed; the weight at P will preponderate and cause the float to sink, making the water rise again until it reaches BD, as before, when the weight and pressure will balance each other. The converse of this is also true, as, if the water be raised above BD, the pressure on DC will be increased, the float will rotate in the other direction, raising it out of the water, and lowering the surface level till it once more reaches BD, and the balance will be restored, the water and the float remaining at rest.

If the semi-cylinder be homogeneous, its centre of gravity will be distant from the axis D, the cube of the radius divided by one and a half times the area of the semicylinder, which, if the radius be considered as 1, is about 0·4244; the distance of P from D is 0·6666 = that of G. A weight at P′ (0·4244 from D), to have an equal force with one at P′ (0·6666), must be inversely as their distances; therefore, the weight of the semicylinder, whose centre of gravity is at P′, is to the weight at P as 0·6666 to 0·4244. And, as the pressure at P, or G, is represented by the weight of a quantity of water equal in bulk to the triangle DCE, the specific gravity of the semicylinder will be inversely as its area to that of the triangle, and directly as their weights, or about one-half the specific gravity of the water. But if the weighting be applied at the circumference, as the centre of gravity of such an arc would be about the distance 0·6366 from D, multiplied by the radius, its aggregate weight should be to the homogeneous semicylinder as 0·4244 to 0·6366, which would be nearly one-third lighter than if homogeneously balanced; and that is the case, no matter what the breadth of the float: but the same does not hold if the figure be a hemisphere, as the centre of gravity of such a body is about 0·375 from its axis, and the centre of gravity of a hemispherical surface is at five-tenths of the radius, which is greater than the ratio given for the semicylinder.

The effect of expansion by heat is not very appreciable, however; if the semicylinder be employed for very accurate

investigations, the relative lengths of the radius and width of semicylinder may be proportioned so as to overcome the effect of difference of expansion of the material of the float and water. In Fig. 2, let BDC be a sector of a hollow metallic cylindrical

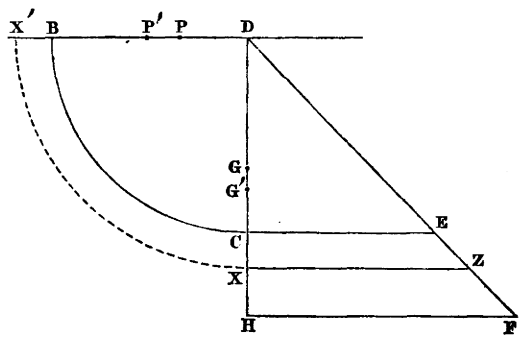

Fig. 2.

float; the triangle DCE represents the volume of water pressing against the side DC, supposed as the force concentrated at G. The rotating force acting on the axis D is the area of the triangle DCE, multiplied by DG, the distance of G from D, or $-\frac{DC^2}{2} \times DG$. Let us suppose the side of the float, and the column of water expanded, to X; the triangle DXZ represents the volume of water, its force concentrated at the point G′, to which G had been expanded; and the rotating force now acting at G′, is $\frac{DX^2}{2} \times DG'$, and as the point P also became P′ by expansion (equal ratio with G′), the balance is maintained; but water expands more than metals by equal increments of heat. Let H represent the point to which the water expanded, and the triangle DHF its volume, which is greater than DXZ, but only equal to it in weight; the rarefied volume of water DXZ, acting on DX′, will not balance the weight acting at P′; but, to keep the semicylinder in equilibrium during changes of temperatnre, the breadth must be so much longer than the

radius, as will, by its expansion, present a surface equal to the difference between the triangles, that is, the breadth must be to the radius as (the difference between the triangles divided by the expanded radius) is to the expansion of the radius.

In balancing the semicylinders, less or more weight may be employed to produce the same effect, provided the centre of gravity is further from the axis, or nearer to it; for instance, if the balancing for a homogeneously balanced semicylinder be placed at its centre of gravity, 0·4244 from the axis, it will be half the specific gravity of water; but if the balance be placed at half that distance, the *whole* weight will equal that of an equal bulk of water, and if at one quarter the distance, it will be double the specific gravity of water. Thus, the same effect is produced by bodies whose absolute weights are so different, that is, they will sink by the withdrawal of water below the surface level, or rise on any addition being made, though one may be much lighter than water, one equal to it, and one double its density. But if the weighting be made to act with a *force* greater than half the specific gravity of water, the power of such excess of weight acts as the whole weight, that is, as the sine of the angle of rotation; and if the whole of the float be elevated to the fluid level, by the withdrawal of a quantity of the fluid, the float will commence to descend, and, in doing so, actually raise the level of the fluid surface, producing the paradox of raising the height of fluid in a vessel by withdrawing a part; but the fluid will continue to rise only while the float is descending through the first quadrant, for, as soon as the point B, in Fig. 1, falls on the line of surface MN, the level will fall, and continue to fall during the further descent of the float through the second quadrant; the converse of this is also true.

The form of the float ought to be that of a figure generated by a plane revolving on its axis. If otherwise, let AEFC represent a parallelopiped, equal in weight to the semi-

cylinder ABC, Fig. 3, and having their centres of gravity coin-

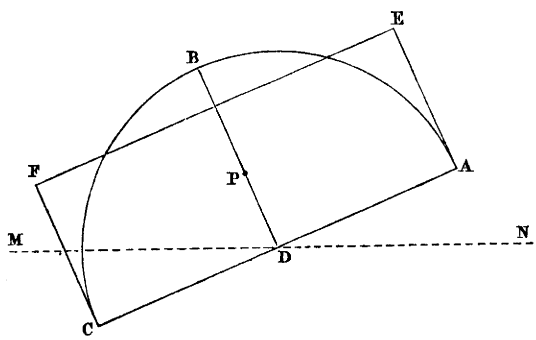

FIG. 3.

ciding. The pressure of the water on DC balances the weight of either of them, considered separately, but the pressure of the water on FC tends to make the parallelopiped rotate in the same direction as that on DC, which tendency not being in the same ratio for the angular motion as that exerted on DC, the two figures cannot act in the same manner, nor can any other figure, where the action on the periphery produces a rotating effect, except on the one given part DC. But a part, or parts, might be excentric in one direction, provided they be counteracted by others in an opposite direction.

The semi-cylindrical float described above is capable of being applied to many useful purposes; the delicacy of its action, when properly balanced, is such, that a solid body capable of raising the surface level in the cistern, in which the compensator may be placed, only 1-2000th of an inch, it will, on being gently plunged into the water, cause an elevation of the float quite visible, and indicating a movement, perhaps, equal to the bulk of the immersed solid. It is, therefore, applicable to the measurement of complicated structures, such as groups of crystals, or masses of other matter, and by it also specific gravities, expansion of solids, &c., may be ascertained with

great exactitude. Extremely small additions of fluid will be measured, as in the instance with the solid already mentioned, and, therefore, its utility as a rain gauge. I also find that, by plunging one end of an open tube, bent at right angles, into the water in the cistern, allowing the wind to act upon the other end,—the surface of the water in the rest of the cistern, and the float, being protected from the influence of the wind,—the float will ascend in exact proportion to the force of the wind, depressing the water in the tube. I have one of these instruments so sensitive that mere breathing or speaking in front of the open end of the tube will act on the float. It has also occurred to us, that a solid cylinder of iron placed in a properly formed cistern of a barometer, would keep the level of the mercury in the cistern constant during the ascent and descent of the mercury in the tube, and would save much trouble in determining the true difference of height of a column in the barometer, which could always be read off at once on the scale without any allowance for difference of level; the surface level of the cistern being maintained to the 1–2500th of an inch by the action of the float.

Since I became acquainted with the properties of the rotating float, my friend, Mr. Richard E. Donovan, who first introduced it to my notice, has informed me that he has recently heard that a similar float had been proposed many years ago as a method for maintaining the oil level in a lamp. However, if that be true, the valuable properties it possesses as a hydrostatic balance could not have been investigated, otherwise it would not have been forgotten. I must here acknowledge the obligations I am under to Mr. Donovan, for the part he took in carrying on the experiments and the calculations necessary for this paper.

The President presented to the Academy, on the part of the present Earl of Charleville, a portrait of his grandfather,

William Bury Earl of Charleville, who was President of the Academy from 22nd June, 1812, to 16th March, 1822.

The President was requested to convey to the Earl of Charleville the special thanks of the Academy for his handsome gift.

Rev. J. H. Jellett, Secretary of the Council, presented the first ten volumes of "Liouville's Mathematical Journal" to the Library of the Academy.

The thanks of the Academy were voted to the Secretary of the Council for his valuable gift.